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An experimental study of end effects for rectangular resonators on narrow channels

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Quarter-wavelength resonators for harbour entrances and similar applications are usually designed and built without regard to the end effect. This paper describes tests on rectangular resonators using water waves of unique frequency in a long narrow wave channel fitted with a wave generator at one end, a wave absorber at the other end and suitable wave filters. In these tests the wavelength was kept constant while the geometry of the rectangular resonant branch canal was systematically varied. Wave transmission across the resonator, as well as wave reflexion upstream of the resonator was measured. The results clearly indicate a considerable effect of main channel width on optimum resonator width and resonator length, invalidating the usual resonance theory (which predicts complete reflexion of the incident wave train when the length of the branch canal is one-quarter of a wavelength). For example, it is found that, under certain conditions, quarter-wavelength resonators may not have *any* effect on the incident wave. The work described is limited to the first resonant mode and to semi-infinite domains on each side of the resonator.

Introduction

A resonator is taken to be a short rectangular branch canal on a narrow main water wave channel. Resonator geometry may be considered to be intermediate in shape between extremes that are either long and narrow, when the motion is in a direction normal to that of the waves in the main channel, or short and wide, when the motion is parallel to that in the main channel (see figure 1). Each of these cases can be handled by one-dimensional wave-propagation theory (James 1968), but the generalized behaviour for intermediate shapes has not been treated mathematically, so far as the writer is aware. (It is hoped that the publication of these results will stimulate interest in a mathematical analysis of the problem.) Valembois (1953) investigated models of short rectangular resonators built orthogonally onto parallel entrance breakwaters of a harbour, but did not observe any end effects. Rayleigh (1929, p. 487) had previously developed an approximation for the end effect of an acoustic tube in a semi-infinite domain, an analogous situation. In this investigation the configuration tested was similar to that of Valembois; the effect of distributed reflexions from the downstream domain was not considered.

In general the dimensions of a resonator should be carefully chosen so that the mass of liquid contained within the resonator and that part of the main channel contiguous with the resonator mouth, i.e. the *junction element* shown in figure 1, has a fundamental frequency equal to that of the incident wave. The incident wave is to be totally reflected back into the upstream domain. For the purposes of this paper the lowest-frequency eigenvalue is used; where a battery of resonators is tuned to cover a wide frequency band, resonance also occurs at higher harmonics.

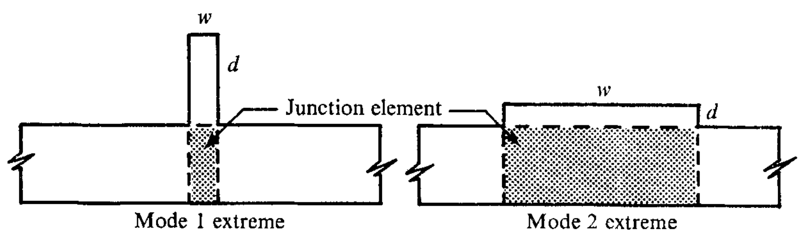

FIGURE 1. Resonator geometry.

Evidently, from the work by Penny & Price (1952), the maximum zone of protection in the downstream domain provided by a resonator at resonance will be limited to a width of one half the incident wavelength. Observations certainly show that wave energy transmitted into the downstream domain increases directly with an increase in major channel width above one wavelength, for the case where resonators are placed on both sides of the main channel. It follows that a resonator investigation of this type can be limited to a major channel width of one wavelength. Le Mehaute (1961) found that wave propagation is one-dimensional at distances of the order of twice the depth from a discontinuity in such narrow channels. This was confirmed in these experiments (except where the width was close to an integral number of half-wavelengths, when a transverse oscillation tended to be set up).

Limiting the treatment to semi-infinite domains results in simpler mathematics (James 1968) but introduces complications into the experiments, since the wave channels should be completely decoupled in both domains. Further mathematical simplification results from the use of first-order linear wave theory, but, for accuracy, non-linear effects in the experimental waves must be accounted for (Goda & Abe 1968). Consequently, in this study wave measurement was effected by micrometers, and the readings were correct to the nearest 0·005 inch.

Typical but exaggerated shapes of the water surface for the first two resonant modes are sketched in figures 2 and 3. Evidently the oscillation differs significantly from that to be expected for quarter-wavelength (i.e. long and narrow) resonators. One consequence of this discrepancy is reported in this note.

Scope of the experiments

The water depth (usually 8·65 in.) and wave period were kept constant throughout each sequence of changes of geometry; reflectivity (ratio of amplitudes of reflected and incident wave trains), transmissivity (ratio of amplitudes of

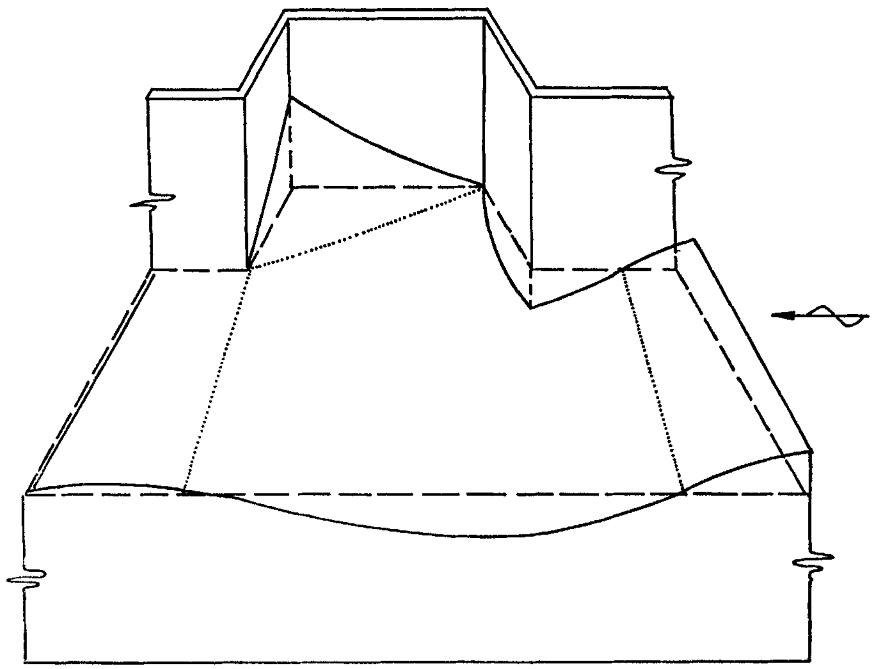

FIGURE 2. First resonant mode.

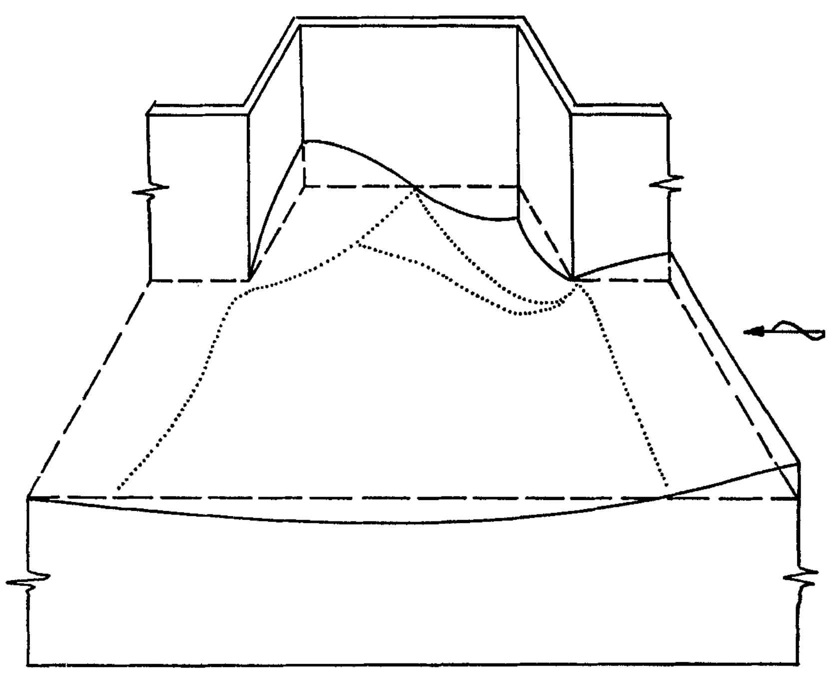

FIGURE 3. Second resonant mode.

transmitted and incident wave trains), and resonator activity were measured for each geometrical configuration tested. The geometries were chosen to give an even spread of results, and to avoid the frequencies resulting in transverse resonance.

All experiments were carried out in a 9 in. wide Perspex wave channel, approximately 50 ft. long, with the resonator housing equidistant from each end. The paddle arrangement was capable of delivering consistent waves of unique period over a sustained time. (A variation in period of 0·001 sec was the maximum tolerated.)

An intermediate Perspex wall was used to vary the width of the main channel, W, between the limits

$$0{\cdot}17 < W/\lambda < 0{\cdot}77,$$

where λ = wavelength.

Adjustable side walls were used to vary the widths of the resonators w for each main channel width tested within the limits

$$0{\cdot}05 < w/\lambda < 0{\cdot}69.$$

The rear walls of the resonators were also adjustable and were clamped sequentially in 8 positions for each resonator width. Resonator geometry tested varied between the limits

$$0{\cdot}23 < w/d < 4{\cdot}7,$$

where d is the length of the resonator. The symbols, w and d are also defined in figure 1. The geometries tested are summarized in table 1.

h	λ	W	w	d
8·65	23·41	18·0	2·0, 4·85, 8·0, 10·0, 12·0, 14·0, 16·0	0 → 13·0
8·65	23·41	9·0	1·0, 2·0, 4·0, 6·0, 8·0, 10·0, 12·0, 14·0	0·5 → 12·5
8·65	29·25	18·0	4·87, 10·0, 18·0	0·75 → 13·0
8·65	52·63	9·0	6·0, 10·0, 18·0	0 → 18·0

TABLE 1. All dimensions are in inches. These tests refer to the data summarized in figure 6

Experimental arrangement

Two 36 in. long Neyrpic wave filters, constructed from 16 gauge perforated zinc plate at 0·5 in. pitch, were used in both domains. Measured transmissivity for each filter varied between 0·4 and 0·6, depending upon wavelength, whence the *decoupling coefficient* for each domain was at worst 0·06, assuming total reflexion off the paddle. The absorber was similar to that constructed by Hamill (1963), and reflectivity was of the order of 0·02. Measurements of reflectivity for the entire domain 2 arrangement demonstrated two points: (*a*) decreasing transmissivity with decreasing wavelength, caused by viscous damping; (*b*) increasing reflectivity with increasing wavelength, caused by the limited dimensions of the filters and absorber. The range of wavelengths tested was carefully limited to minimize both these effects, but, for the viscous damping effect, corrections were applied to all results.

Wave measurement

Where reflectivity is to be measured to 1 %, envelope heights should be measured to 0·5 %. By using fine probes, and limiting instantaneous contact to a depression of the crest surface rather than actually puncturing it, it is usually impossible to see any meniscus effect. Hence sharpened stainless steel probes, nominal diameter about 0·015 in., were connected to an oscilloscope with a stationary time base. Dispersive agents further reduced surface-tension effects and resulted in clear signals being relayed to the cathode ray oscilloscope.

As a check on the consistency of the experimental waves, standard deviations were measured using a dekatron counting unit. The count recorded the 'number of crests greater than' (or 'number of troughs less than') for each setting of the micrometers attached to the probes. Batches of 100 consecutive waves for

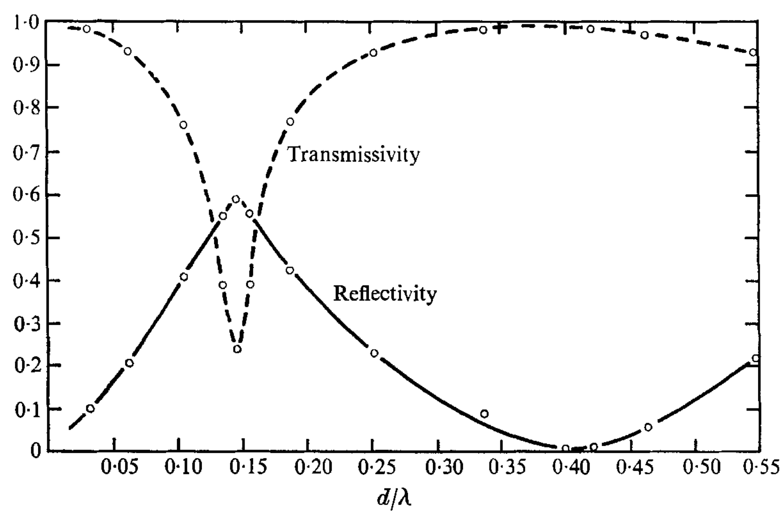

FIGURE 4. Typical results for narrow resonators. $w/\lambda = 0{\cdot}34$, $W/\lambda = 0{\cdot}77$, $2a/\lambda = 0{\cdot}049$.

micrometer increments of 0·0005 in. were counted, and histograms of the crest and trough relative frequency plotted. Most were approximately symmetrical, and computed standard deviations were of the order of 0·0015 in. for incident wave amplitudes of 0·25 in. Hence the mean reading was given by approximately half the number of a fairly large sample of waves, say 20. An analysis of the scatter on the curves with a high density of plotted points eventually indicated a random error of the order of 1 %.

Results

Healy's method (Goda & Abe 1968), also known as the loop-and-mode method (James 1969), was used to compute reflectivity and incident wave amplitudes. Since this method is based upon assumptions of linear theory and of only two wave components being present in the upstream domain, corrections had to be applied (Goda & Abe 1968). Results for each width of resonator were plotted as shown in figures 4 and 5. Corrections for attenuation, downstream reflectivity and non-linearity do not affect the geometry for resonance. The first peak in figure 5 applies to the first mode (figure 2), and the second (flatter) peak, the second mode of resonance (figure 3).

From the first maxima and minima of all these plots the family of curves in figure 6 was derived. The end effect is given by $(d/\lambda - 0{\cdot}25)$.

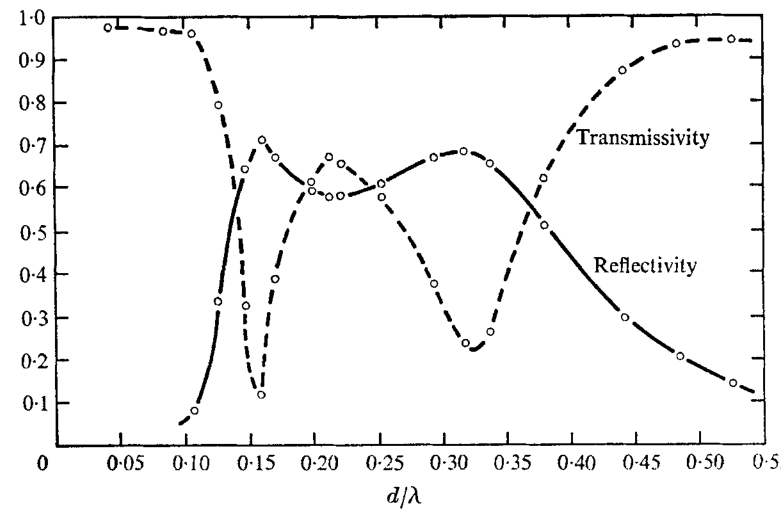

FIGURE 5. Typical results for wide resonators. $w/\lambda = 0{\cdot}60$, $W/\lambda = 0{\cdot}77$, $2a/\lambda = 0{\cdot}017$.

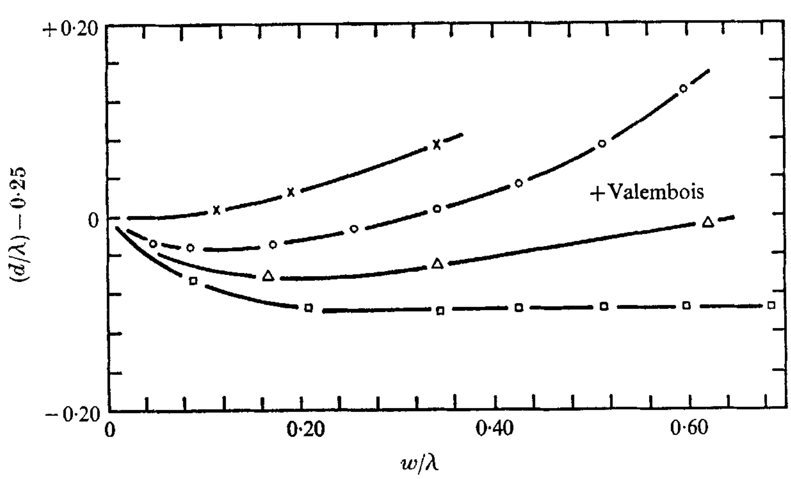

FIGURE 6. Geometry for resonance. Values of W/λ: ×, 0·17; ○, 0·38; △, 0·62; □, 0·77.

A number of tests were also carried out to investigate the effect of water depth, and it was found that geometry for resonance was not affected by such changes. Further tests also demonstrated that geometry for resonance was unaffected by incident wave amplitudes. Transmissivity minima and reflectivity maxima, were, however, sensitive to amplitude changes.

Conclusions

Resonator geometry for resonance is independent of still-water depth, wave amplitude, and energy dissipation in the resonator, and is given by figure 6.

It is clearly seen on figure 5 that the transmissivity for $d/\lambda = 0{\cdot}25$ is sufficiently high ($\beta = 0{\cdot}93$) to render the resonator ineffective. (For this case $W/\lambda = 0{\cdot}77$ and $w/\lambda = 0{\cdot}34$.)

The end effect for narrow resonators (say $w/d < \frac{1}{3}$), and for wide channels is negative, as shown on figure 6. This accords with the work done by Rayleigh.

From the same figure we see that for narrow entrance channels (e.g. $W/\lambda < \frac{1}{3}$) the end effect is generally positive, and that this effect increases with width of resonator. The geometry recommended by Valembois is indicated by a cross, and evidently is suited to a main channel width of approximately $0{\cdot}5\lambda$.

Figure 6 indicates that, for minimum resonator lengths, resonator and main channel widths should not be narrow. Such narrow widths in any case result in higher velocities and concomitant decreased efficiency.

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